ISC MATHS PREPARATORY EXAMINATION 2024

MARKING SCHEME

Q1		
(i) (d)	$x^2 - 2$	A1 (application)
(ii) (c)	$\frac{\pi}{2}$	A1
(iii) (a)	Always continuous	A1 (knowledge)
(iv) (b)	$\begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$	A1 (application)
(v) (d)	$(-\sqrt{3}, \sqrt{3})$	A1 (skill)
(vi) (b)	64	A1 (understanding)
(vii) (d)	$\frac{1}{x} + \frac{1}{x^2}$	A1 (knowledge)
(viii) (b)	Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).	A1 (knowledge)
(ix) (a)	1	A1 (skill)
(x) (b)	$x = -2$	A1 (understanding)
(xi)	true or correct	A1 (application)
(xii)	$X = \frac{1}{2}\left[\begin{pmatrix} 1 & 2 \\ 5 & 6 \end{pmatrix} + \begin{pmatrix} 1 & 5 \\ 2 & 6 \end{pmatrix}\right]$ $= \begin{pmatrix} 1 & \frac{7}{2} \\ \frac{7}{2} & 6 \end{pmatrix}$ $\lvert X \rvert = -\frac{25}{4}$	A1 (application and skill)
(xiii)	$xe^x - e^x = xe^x + k$ $=> k = -e^x$	A1 (Knowledge and application)
(xiv)	Prob. $= \frac{1}{270725}$	A1 (Application)
(xv)	$(0.3 \times 0.4) + (0.7 \times 0.6)$ $= 0.54 = 54\%$	A1 (application)
Q2 (i)	$1 = \left(1 - 2\frac{dy}{dx}\right)e^{x-2y}$ $\frac{dy}{dx} = \frac{1}{2}(1 - e^{2y-x})$ Or $\frac{1}{2}\left(1 - \frac{1}{x}\right)$	M1 A1

Q2 (ii)	$\frac{dy}{dx} = 2x = 2$ $=> x = 1\ , y = -1$	M1 A1
Q3	Using any relevant property Using second property leading to the right side	M1 A1
Q4	$f^{-1}(x) = \frac{5+2x}{x}$ Domain of f^{-1} is range of f and vice versa Domain is $R - \{0\}$ Range is $R - \{2\}$	M1(knowledge) A1 (application)
Q5	$y = \frac{x^2}{4} + \frac{e^x}{2} + (c)$	A1 for one correct integration A2 for both integrations correct
Q6 (i)	Using appropriate property Getting answer $\frac{\pi}{4}$	M1 A1 (Knowledge and application)
Q6 (ii)	$breaking\ into\ two\ integrals$ $= 25$	M1 A1 (application)
Q7	Using Integration by parts $y = vx\ \ , \frac{dy}{dx} = v + x.\frac{dv}{dx}$ $\int \frac{2v}{3-v^2} dv = \int \frac{dx}{x}$ $-\ln\left(3 - \frac{y^2}{x^2}\right) = \ln\left(\frac{y}{x}\right) + C$	M1, A1 M1 A1
Q8	$\tan^{-1}\left(\frac{x-2}{x-4}\right) + \tan^{-1}\left(\frac{x+2}{x+4}\right) = \frac{\pi}{4}$ $\tan^{-1}\left(\frac{\frac{x-2}{x-4} + \frac{x+2}{x+4}}{1 - \frac{x-2}{x-4}.\frac{x+2}{x+4}}\right) = 1$ $=> 2x^2 - 16 = -12$ $=> x^2 = 2$ $=> x = \pm\sqrt{2}$	M1 A1 M1 A1

Q9 (i)	Equation of any line parallel to the given line is $4x - y + C = 0$ Slope of the line is 4 Slope from derivative $= 3x^2 - 4x$ $=> 3x^2 - 4x = 4$ $=> x = 2, -\frac{2}{3}$ Corresponding $y = 6, \frac{130}{27}$ So, equations are (i) $y - 6 = 4(x - 2)$ (ii) $y - \frac{130}{27} = 4(x + \frac{2}{3})$	M1 A1 A1 A1
Q9 (ii)	$A = \pi r^2$ $\frac{dA}{dt} = 2\pi r.\frac{dr}{dt}$ $14\pi \times 4$ $= 14 \times \frac{22}{7} \times 4$ $= 176\ {}^{m^2}/_{sec}$	M1 A1 M1 A1
Q 10 (i)	$S = \{(H,H),(H,T),(T,1),(T,2),(T,3),(T,4),(T,5),(T,6)\}$ $A = \{(T,5),(T,6)$ $B = \{(T,1),(T,2),(T,3),(T,4),(T,5),(T,6)\}$ $Required\ probability\ = P(A/B) = \frac{P(A \cap B)}{P(B)}$ $= \frac{2}{6}$ $= \frac{1}{3}$	M1, A1 for any two substitutions correct, M2 for all substitutions correct M1 for final answer
Q 10 (ii)	$P\left(\frac{H}{Grade\ A}\right) = \frac{P(H)P\left(\frac{Grade\ A}{H}\right)}{P(H)P\left(\frac{Grade\ A}{H}\right) + P(D)P\left(\frac{Grade\ A}{D}\right)}$ $= \frac{\frac{60}{100} \times \frac{30}{100}}{\frac{60}{100} \times \frac{30}{100} + \frac{40}{100} \times \frac{20}{100}}$	M1, A1 for any one substitution correct, M2 for all substitutions correct M1 for final answer

	$= \frac{18}{18+8}$ $= \frac{18}{26}$ $= \frac{9}{13}$	
Q 11	$\begin{pmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{pmatrix} \begin{pmatrix} 1/x \\ 1/y \\ 1/z \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$ $\begin{pmatrix} 1/x \\ 1/y \\ 1/z \end{pmatrix} = \begin{pmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{pmatrix}^{-1} \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$ $\Rightarrow \begin{pmatrix} 1/x \\ 1/y \\ 1/z \end{pmatrix} = \frac{1}{1200} \begin{pmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$ $x = 2, y = 3, z = 5$	M1 A1 for determinant correct A2 for all cofactors correct, A1 for any six cofactors correct A2 for all three correct, A1 for two correct
Q12 (i)	(a) $\pi r^2 h = 20\pi$ $\Rightarrow h = \frac{20}{r^2}$ (b) $C = 2(2\pi r^2) + 5(2\pi r h)$ $= 4\pi r^2 + \frac{200\pi}{r}$ (c) $\frac{dC}{dr} = 8\pi r - \frac{200\pi}{r^2}$ $\Rightarrow r = \sqrt[3]{25}$ $C_{min} = 4\pi\left(\sqrt[3]{25}\right)^2 + \frac{200\pi}{\sqrt[3]{25}}$ $= 12\pi\left(\sqrt[3]{25}\right)^2$	A1 M1,A1 A1 A1 A1

Q12 (ii)	*Let side of the square to be cut = x cm* Volume : $V = (45-2x)(24-2x)x$ $$V = 4x^3 - 138\,x^2 + 1080x$$ $$\frac{dv}{dx} = 12x^2 - 276x + 1080$$ $$12x^2 - 276x + 1080 = 0$$ $$=> x = 18\ and\ 5$$ $x = 18\ discarded$ as it is point of minimum $$Vmax = (45-10)(24-10)10 = 4900cm^2$$	M1 M1 A1 M1 R1 A1
Q 13 (i)	$$\frac{2xdx}{x+1} = \frac{dy}{y}$$ $$=> 2(x - \ln(x+1)) = \ln y + c$$ Replacing x and y values $$c = 4 - 3ln3$$ $$=> 2(x - \ln(x+1)) = lny + 4 - 3ln3$$	M1 A1 for one integration correct, A2 for both correct M1 A1 A1
Q 13 (ii)	$$\int \frac{x^2}{(x^2-2)(x^2+4)}dx$$ By breaking into partial fractions $$\frac{1}{3}\left[\int \frac{1}{(x^2-2)}dx + \int \frac{2}{(x^2+4)}dx\right]$$ $$= \frac{1}{3}\left[\frac{1}{4}ln\left(\frac{x-2}{x+2}\right) + \tan^{-1}\frac{x}{2}\right] + c$$	M1,A2 for breaking into partial fraction correctly A1 for one integral correct, A3 for fully correct.

<table>
<tr>
<td>Q 14</td>
<td>
<table>
<tr><th>x</th><th>$P(x)$</th><th>$x.P(x)$</th></tr>
<tr><td>0</td><td>$\frac{26.25.24.23}{30.29.28.27}=\frac{2990}{5481}$</td><td>0</td></tr>
<tr><td>1</td><td>$4\times\frac{26.25.24.4}{30.29.28.27}$
$=\frac{2080}{5481}$</td><td>$\frac{2080}{5481}$</td></tr>
<tr><td>2</td><td>$6\times\frac{26.25.4.3}{30.29.28.27}=\frac{130}{1827}$</td><td>$\frac{260}{1827}$</td></tr>
<tr><td>3</td><td>$4\times\frac{26.4.3.2}{30.29.28.27}=\frac{104}{27405}$</td><td>$\frac{312}{27405}$</td></tr>
<tr><td>4</td><td>$\frac{4.3.2.1}{30.29.28.27}$
$=\frac{1}{27405}$</td><td>$\frac{4}{27405}$</td></tr>
</table>
Mean $= 0.53$
</td>
<td>M1 for any two entries correct in column 1
A2 for all entries correct in column 2

M2 for any two entries correct in column 3

A1</td>
</tr>
<tr>
<td>Q15</td>
<td>(i) c

(ii) b

(iii) $\frac{5}{\sqrt{21}}\ (2\hat{\imath}-\hat{\jmath}+4\hat{k})$

(iv) $y=5$

(v) $\frac{6\hat{\imath}-6\hat{\jmath}+9\hat{k}}{3}$</td>
<td>A1

A1

A1

A1

A1</td>
</tr>
<tr>
<td>Q 16
(i)</td>
<td>$(\vec{r}\times\hat{\imath})^2$
$\left((x\hat{\imath}+y\hat{\jmath}+z\hat{k})\times\hat{\imath}\right)^2$
$=\left(-y\hat{k}+z\hat{\jmath}\right)^2$
Showing dot product leading to final result</td>
<td>A1

A1</td>
</tr>
<tr>
<td>Q16
(ii)</td>
<td>$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})$
$=(4\hat{\jmath})\times(2\hat{\imath}+4\hat{k})$
$=16\,\hat{\imath}-8\hat{k}$
Unit vector is $\frac{1}{\sqrt{320}}(16\,\hat{\imath}-8\hat{k})=\frac{1}{\sqrt{5}}(2\,\hat{\imath}-\hat{k})$</td>
<td>A1

A1</td>
</tr>
<tr>
<td>Q 17
(i)</td>
<td>Shortest distance $(a_1-a_2).\frac{(b_1\times b_2)}{|b_1\times b_2|}$
$b_1\times b_2=\begin{pmatrix}4\\6\\8\end{pmatrix}$
$a_1-a_2=\begin{pmatrix}4\\6\\8\end{pmatrix}$
$S.D=\frac{16+36+64}{\sqrt{116}}=\sqrt{116}$</td>
<td>A1

A1

M1,A1</td>
</tr>
</table>

Q 17 (ii)	$P_1: 2x + 6y + 12 = 0$ $P_2: 3x - y + 4z = 0$ Required plane is $(2x + 6y + 12) + \lambda(3x - y + 4z) = 0$ Replacing the point $(1,1,1)$ Finding $\lambda = -\frac{10}{3}$ Final answer $-6x + 7y - 10z + 9 = 0$	M1 M1 A1 A1
Q 18	Finding x coordinates of points of intersection as $2, \frac{7}{2}$ Area $= \int_2^{\frac{7}{2}}[-3(x-3)^2 + 4 - (x-2)^2 - 1]dx$ $= \left[-\frac{4}{3}x^3 + 11x^2 - 28x\right]_2^{\frac{7}{2}}$ Final Answer =2.25	A1 M1, A1 for correct substitution A1 for substituting the limits correctly. Final answer in simplified for is not required.
Q 19	(i) d (ii) c (iii) Demand function $= -\frac{2b}{x}$ (iv) $r = \sqrt{\frac{3}{22}}$ (v) $M.C = \frac{-1}{x^2}$	A1 A1 A1 A1 A1
Q 20 (i)	$R(x) = C(x)$ $\Rightarrow 4000x = 500x^2 + 1500x - 3000$ $\Rightarrow x = 6$	M1 A1
Q 20 (ii)	$A.C = \frac{a}{x} + b + cx$ $\Rightarrow \frac{d}{dx}(A.C) = -\frac{a}{x^2} + c$ Showing that the other side also has the same value	A1 A1
Q 21	$b_{xy} = \frac{3}{2}, b_{yx} = \frac{1}{3}, r = \frac{1}{\sqrt{2}}$ $b_{yx} = r.\frac{\sigma_y}{\sigma_x}$ $\sigma_y^2 = \frac{10}{9}$	A1 M1, A1 (replacing values in this) A1

<table>
<tr>
<td>Q 22
(i)</td>
<td>x numbers of fans
y numbers of sewing machine

Profit $= 220x + 180y$

Constraints $3600x + 2400y \leq 57600, x + y \leq 20, x, y \geq 0$

<table>
<tr><th>Corner points</th><th>Output $= 220x + 180y$</th></tr>
<tr><td>$(0,0)$</td><td>0</td></tr>
<tr><td>$(8,12)$</td><td>3920</td></tr>
<tr><td>$(16,0)$</td><td>3520</td></tr>
<tr><td>$(0,20)$</td><td>3600</td></tr>
</table>

Maximum profit of Rs.3920 when 8 fans and 12 sewing machines are purchased</td>
<td>A1

A1 if two correct, A2 if three correct

A1</td>
</tr>
<tr>
<td>Q 22
(ii)</td>
<td>Rice : x bags
Wheat: y bags
Profit $= 11x + 9y$
Constraints $x + y \leq 10, 3x + 2y \leq 25, x \geq 0, y \geq 0$
<table>
<tr><th>Corner points</th><th>Profit$= 40x + 50y$</th></tr>
<tr><td>$(0,0)$</td><td>0</td></tr>
<tr><td>$(0,10)$</td><td>90</td></tr>
<tr><td>$(5,5)$</td><td>100</td></tr>
<tr><td>$\left(\frac{25}{3}, 0\right)$</td><td>91.67</td></tr>
</table>
Max profit $= ₹100$ when 5 bags of rice and 5 bags of wheat are purchased.</td>
<td>A1
A1 for two correct, A2 for all correct

A1</td>
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